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Parabolae. (See Fig. VII). In order to find the parabolae through two given points P and Q, having a given focus S, we need to find the directrices. The directrices of the parabolae through P are evidently the tangents to the circle of radius SP and center P, and those of the parabolae through Q, the tangents to the circle of radius SQ and center Q. Hence the parabolae through P and Q have the common tangents of these two circles as directrices. These tangents will always be real, and distinct, unless P, Q, and S lie on the same straight line and P and Q lie on the same side of S. Then there is only one common tangent, one that passes through S, and hence in this case the parabolae become coincident and degenerate into the straight line PQS. There is only one other exceptional case, when P, Q, and S lie on the same line and P and Q lie on opposite sides of S. Then there will be three common tangents, and there will consequently be three real parabolae through P and Q. One of these is the straight line PQS.

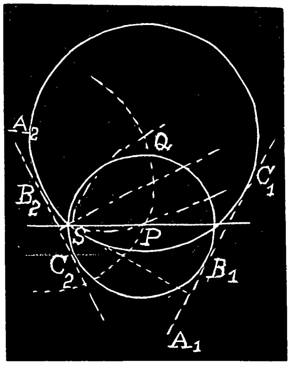

Fig. VII.

The parabolae through a point P with focus S have no envelope.

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DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

ALGEBRA.

295. Proposed by CHARLES GILPIN, JR., Philadelphia, Pa.

In the equation $x^3-ax\pm b=0$, we have the following relation between the coefficients and the roots: (1) When $a^3/b^2=6.75$ there are three real roots, two of which are equal; (2) when $a^3/b^2<6.75$ there are two imaginary roots and one real one; and (3) when $a^3/b^2>6.75$ there are three real, unequal roots.

Remarks and Solution by G. B. M. ZERB, A. M., Ph. D., Philadelphia, Pa., and H. V. SPUNAR, East Pittsburg, Pa.

The proposer has interchanged cases (2) and (3). When $a^3/b^2<6.75$, there are three real unequal roots; when $a^3/b^2>6.75$ there are two imaginary and one real root.

Let $m^3=\pm\frac{1}{2}b+\sqrt{\frac{b^2}{4}-\frac{a^3}{27}}$, $n^3=\pm\frac{1}{2}b-\sqrt{\frac{b^2}{4}-\frac{a^3}{27}}$, and ω and ω^2=the cube roots of unity. Then the roots are $m+n$, $\omega m+\omega^2 n$, $\omega^2 m+\omega n$. When $b^2/4=a^3/27$, or $a^3/b^2=\frac{27}{4}=6.75$, $m=n=\pm\frac{1}{2}\sqrt[3]{(4b)}$.

$\therefore m+n=\pm\sqrt[3]{(4b)}$, $\omega m+\omega^2 m=\mp\frac{1}{2}\sqrt[3]{(4b)}$, $\omega^2 m+\omega n=\mp\frac{1}{2}\sqrt[3]{(4b.)}$

When $b^2/4<a^3/27$, or $a^2/b^2<\frac{27}{4},=6.75$, $\sqrt{(b^2/4-a^3/27)}$ is imaginary.

Let $m=u+\sqrt{(-1)}v$, $n=u-\sqrt{(-1)}v$.

$\therefore m+n=u+v$, $\omega m+\omega^2 n=-u-v\sqrt{3}$, $\omega^2 m+\omega n=-u+v\sqrt{3}$, all real and unequal.

When $a^3/b^2>\frac{27}{4},=6.75$, $\sqrt{\frac{b^2}{4}-\frac{a^3}{27}}$ is real.

$\therefore m+n$ is real, and $\omega m+\omega^2 n$, $\omega^2 m+\omega n$ are imaginary.

J. W. Clawson, of Collegeville, Pa., referred to Burnside and Panton's *Theory of Equations*, Vol. I, §§42, 43. Discussions of this problem are to be found in nearly all texts on the Theory of Equations.

296. Proposed by G. B. M. ZERR, A. M., Ph. D., 4243 Girard Avenue, Philahelphia, Pa.

Sum the series, $1+\frac{1}{6}+\frac{1}{20}+\frac{1}{50}+\frac{1}{105}+\frac{1}{196}+\frac{1}{336}+\ldots$

Solution by A. R. MAXSON, A. M., Columbia University, New York.

In the series 1, 6, 20, 50, 105, 196, 336, ..., the successive orders of differences are,

$$\begin{array}{c} 5,\ 14,\ 30,\ 55,\ 91,\ 140,\ \ldots \\ 9,\ 16,\ 25,\ 36,\ 49,\ \ldots \\ 7,\ 9,\ 11,\ 13,\ \ldots \\ 2,\ 2,\ 2,\ \ldots \\ 0,\ 0,\ \ldots \end{array}$$

The nth term is then

$$1+5(n-1)+\frac{9}{2\,!}(n-1)(n-2)+\frac{7}{3\,!}(n-1)(n-2)(n-3)$$

$$+\frac{2}{4\,!}(n-1)(n-2)(n-3)(n-4)=\frac{n}{12}(n+2)(n+1)^2.$$

The nth term of the given series is then $\frac{12}{n(n+2)(n+1)^2}$, which can be written $\left(\frac{6}{n}+\frac{6}{n+1}\right)-\left(\frac{6}{n+1}-\frac{6}{n+2}\right)-\frac{12}{(n+1)^2}$.

Taking now u_r as the rth term of the original series, we have